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# NONLINEAR TRANSIENT SHOCK RESPONSE ANALYSIS FOR SUBMERGED SHELL STRUCTURES

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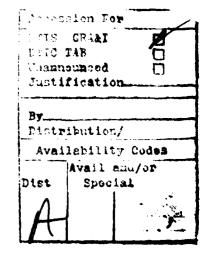
linear transient response analysis of structures. The acceleration residue flexibility procedure, which incorporates the static response at each time, is coupled to the modal superposition method. The nonlinearities are taken into account by using updated stiffness information in the static response. The effectiveness of the method is derived from the use of efficient reanalysis techniques to calculate the static solutions.

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# SUMMARY

This report proposes the use of a modal superposition approach for the nonlinear transient response analysis of structures. The acceleration or residue flexibility procedure, which incorporates the static response at each time, is coupled to the modal superposition method. The nonlinearities are taken into account by using updated stiffness information in the static response. The effectiveness of the method is derived from the use of efficient reanalysis techniques to calculate the static solutions.





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## SECTION I INTRODUCTION

Structural damage to submerged shell structures excited by a shock wave is an inherently nonlinear phenomenon. Fundamental to the study of such phenomena is the formulation of a consistent and reliable theory to predict the nonlinear transient response of a shell structure. Naturally, a complete investigation of the problem must include the fluid-structure iteration. Here only the nonlinear structural response will be considered.

The modal superposition method has for many years been the dominant dynamic analysis method for linear systems. Recently, it has been suggested [Ref. 1,2,3] that the modal superposition method can be successfully applied to nonlinear problems. It would appear that modal superposition is competitive with direct integration for many nonlinear problems and may be better for problems involving "wide-band" governing equations.

It has also been recognized that the modal superposition method is aided when it is supplemented with a static solution calculated for the load applied at particular times. In dynamics treatises of 20 or 30 years ago, this method was referred to by a variety of names, e.g., the <u>acceleration method</u>. It is now sometimes called the <u>residue flexibility</u> method. This method was recognized as improving convergence, at least during the period for which the loading is applied. It is now believed [Ref. 4] that the use of the static response will account for the effect of higher modes.

## SECTION II FORMULATION OF THE METHOD

## 1. DEVELOPMENT OF THE METHOD

It is proposed that a modal superposition analysis be employed for nonlinear problems. The static response at each point in time is to be used to supplement the modal summation. The nonlinearities can be incorporated using an incremental or a pseudo-force approach. For the formulatioin to be outlined here, use of the pseudo-force method is probably the most straight forward technique.

Let the p normal mode shapes for the structure at time t = 0 be

$$\{\phi_1\}$$
  $\{\phi_2\}$  . . .  $\{\phi_p\}$  (1)

These should be computed at the outset of the solution.

At each time t calculate the static displacements due to the load occurring at time t. Use the updated stiffness characteristics. Let  $\{u_{NL}\}$  be this solution, i.e. the solution to the static problem at time t.

Let {u} be the physical degrees of freedom. Then, set

$$\{u\} = \{\phi : u\} \{q\} = \{R\} \{q\}$$
 (2)

where

$$[\phi] = [\{\phi_1\} \ \{\phi_2\} \ . \ . \ ]$$
 (3)

{q} = modal coordinates

$$[R] = \{ \phi : \mathbf{u} \}$$

$$\mathbf{NL}$$

The modal equations would then be coupled

$$T$$
  $[R] [R] [q] + [R] [k] [R] [q] = [R] [F] (5)$ 

where

[m] is the mass matrix

[k] is the stiffness matrix

{F} is the loading vector

Although Eqs. (5) are coupled, note that

$$[R]$$
  $[m]$   $[R]$  and  $[R]$   $[k]$   $[R]$ 

are of much smaller order than [m] and [k]. In fact, for linear systems they take the special form

For some cases, an efficient algorithm for solving Eq. (5) can probably be established.

As an alternative to this procedure, the  $\{u_{NL}\}$  can be transformed to a more favorable set of coordinates. Compute  $\{u_{NL}\}$  from  $\{u_{NL}\}$  which is orthogonal to  $\{\phi_1\},\ldots,\{\phi_p\}$ . Set

$$\{u\} = [\phi : \overline{u}_{NL}]\{q\} = [\overline{R}]\{q\}$$
 (6)

The modal equations will be uncoupled here. However, in order to achieve the uncoupled modal equations it is necessary to calculate  $\{\overline{u}_{NL}\}$  from  $\{u_{NL}\}$ .

# 2. DISCUSSION

The proposed procedure is a nonlinear version of the residual flexibility or acceleration approach for modal superposition. The advantage in this technique over the use of updated  $\{\phi_i\}$  may lie in both greater efficiency and accuracy. There is some question as to the effectiveness of the use of updated mode shapes, say  $\{\psi\}$ , since  $\{\psi\}$  will be formed of linear combinations of  $\{\phi_i\}$ .

To achieve the desired efficiency and accuracy, it is proposed that a novel static reanalysis procedure be employed to update the static solutions at time t. This is the key to the solution technique.

## 3. STATIC REANALYSIS METHODS

The success of the proposed method depends on the availability of an efficient and reliable method for the static reanalysis of a structural system. Before developing such a method we will review some of the currently available reanalysis techniques. The goal of reanalysis technology is to analyze a modified structure by taking advantage of information from the solution to the original or unmodified structure to avoid a complete analysis of the modified structure.

The methods will be grouped as approximate and exact techniques. The approximate methods can be categorized as: perturbation (Taylor series) method, iteration method, and reduced basis method. These are generally applicable to small magnitude modifications distributed over a large portion of the structural system. In constrast, the exact methods are applicable to localized modifications with arbitrary magnitudes. Application of the exact method requires that the structural configuration and the validity of the finite element model are not affected by the modification.

The underlying principles of the exact methods involve the conception of parallel elements, initial stress, and pseudo loads. Pure algebraic formulas such as the Sherman-Morrison formula (5) and its generalization, the Woodbury formula (5) can also be directly used to develop exact modification methods. In this section, a direct method based on the Woodbury formula due to Argyris and Roy (6), a successive modification method proposed by Kavlie and Powell (7) and the pseudo load method developed by the authors are discussed. The last method although similar to that of Argyris and Roy is derived by only using the system's linearity.

In addition to reanalysis the pseudo load approach provides closed (or semi-closed) form relationships between static response variables and the design parameters. This enables the design (i.e., select design parameters) for prescribed responses.

#### Notation

Static Reanalysis

 $\underline{\mathbf{b}} = \mathbf{a}$  Boolean matrix which expands  $\Delta \underline{\mathbf{K}}$  into  $\underline{\Delta \mathbf{K}}$ ,

$$\Delta K = b^T \Delta K b$$

B = system bandwidth

d; = ith 'esign par: meter

D = number : dc. ign variables

K = system stiffness matrix

 $K_i$  = modified system stiffness matrix with the 1 through ith non-zero columns of  $\Delta K$  included

 $\Delta K$  = stiffness modification matrix of system

 $\Delta K = \Delta K$  with all null columns removed

 $\Delta \underline{X}_i = \underline{X}_i = ith non-null column of <math>\Delta \underline{K}$ 

 $\Delta K = \Delta K$  with all null rows removed

L = lower triangular matrix

 $n_c = \text{number of non-null columns (rows) of } \Delta K$ 

P = external load vector

r = number of reduced basis vectors

 $\underline{R}$  = transformation matrix

 $\underline{\underline{U}} = \text{original system solution}$ 

 $\underline{\underline{U}} = modified system solution$ 

 $\underline{\underline{U}}^{(i)} \approx ith basis vector used in the reduced basis method$ 

 $\underline{U}_i$  = modified system solution after the 1 through ith non-null columns of  $\underline{\Delta K}$  are included

 $\underline{W}_{i}^{(k)} \approx \underline{K}_{k}^{-1}\underline{X}_{i} = \underline{K}_{k}^{-1}\underline{\Delta K}_{i}$ 

 $\underline{X}_i = \underline{\Delta K}_i$ , ith column of  $\underline{X}$ 

 $\underline{\underline{Y}}_{i}^{T} \approx \text{ ith row of } \underline{\underline{Y}}^{T}$ 

# Approximate Methods

Taylor Series Expansion (8)

The static responses can be expanded in Taylor series as functions of the design parameters, for example,

$$\underline{\overline{U}} = \underline{U} + \Sigma \xrightarrow{D} \Delta d_{i} + \Sigma \Sigma \xrightarrow{D} \Delta d_{i} \Delta d_{j}$$

$$i=1 \ \partial d_{i} \qquad i=1 \ j=1 \ \partial d_{i} \partial d_{j}$$
(7)

+ higher order terms

The derivative terms are evaluated by differentiating the system's static equilibrium equation, KU = P (8)

assuming that the external load vector's,  $\underline{P}$ , change is negligible when a design parameter changes. The first two derivatives are expressed by,

$$\frac{\partial \underline{u}}{\partial d_i} = -\underline{K}^{-1} \frac{\partial \underline{K}}{\partial d_i} \underline{u} \tag{9}$$

$$\frac{\partial^2 U}{\partial d_i \partial d_j} = -\underline{K}^{-1} \left( \frac{\partial^2 \underline{K}}{\partial d_i \partial d_j} \underline{U} + \frac{\partial \underline{K}}{\partial d_i} \frac{\partial \underline{U}}{\partial d_j} + \frac{\partial \underline{K}}{\partial d_j} \frac{\partial \underline{U}}{\partial d_i} \right) \tag{10}$$

Note that in practice  $\underline{K}$  is not inverted, instead the triangularized form of  $\underline{K}$  obtained from the unmodified system's solution is used to solve for the vectors in equations (9) and (10). In addition, to maximize the computational efficiency, advantage should be taken of the sparsity of the derivatives of  $\underline{K}$ . Ordinarily only the first order terms of equation (7) are retained.

Storaasli and Sobieszczanski (9) have applied the first order Taylor series method to a 5000 dof fuselage midsection model. Less than 16% error was observed for single element variation of -100% (i.e. element removal) to +500%, and for -50% to 50% change for multi-elements. The incorporation of second order terms would undoubtedly increase this solution accuracy, but at the expense of increased computational burden.

Iteration Method

The equilibrium equation of the modified system may be written as,

$$\underline{K} \ \underline{\hat{U}} = \underline{P} - \underline{\Delta K} \ \underline{\hat{U}} \tag{11}$$

From this form an iterative solution approach is derived in the following recursive form,

$$\underline{K} \, \underline{\overline{U}}^{(i+1)} = \underline{P} - \underline{\Delta} \underline{K} \, \underline{\overline{U}}^{(i)} \tag{12}$$

Back substitution is employed to solve for  $\underline{\tilde{U}}^{(i+1)}$ , utilizing the triangularized form of  $\underline{K}$  from the unmodified system's solution. The iteration is started using,

$$\underline{\underline{U}}^{(o)} = \underline{\underline{U}} = \text{unmodified system response}$$
 (13)

Noor and Lowder  $(\frac{7}{2})$  suggested a method comprised of the Taylor series and iteration formulas. First  $\underline{\tilde{v}}^{(0)}$  is computed via a Taylor series method, i.e.

$$\underline{\underline{U}}^{(o)} = \underline{U} + \sum_{i=1}^{D} \frac{\partial \underline{U}}{\partial d_{i}} \Delta d_{i}$$
 (14)

Then  $\bar{\mathbb{U}}^{(0)}$  is used to initiate the iterative process in eq.(12), for i>0. Usually one or two iteration cycles are reported sufficient to obtain good results.

Reduced Basis Method

Fox and Miura (10) introduced a procedure analogous to assumed modes to reduce the dimension of the modified structure's static equilibrium problem. A set of known system deflection vectors  $\underline{\mathbf{U}}^{(1)}$ ,  $\underline{\mathbf{U}}^{(2)}$ , ...,  $\underline{\mathbf{U}}^{(r)}$  are assumed to span the solution space. Since r<<n = number of dof, this is obviously an approximation. The approximate solution of the modified system then follows as,

$$\underline{\underline{U}} = \sum_{j=1}^{r} c_j \underline{\underline{U}}^{(j)}, \quad \underline{c} = [c_1 \ c_2 \ \dots \ c_r]^T$$
(15)

$$\frac{\tilde{K}}{\tilde{K}} = \frac{\tilde{P}}{\tilde{P}}$$
 (rx1)

$$\frac{\tilde{K}}{K} = R^{T}(K + \Delta K) R \qquad (rxr) \qquad (17)$$

$$\underline{\underline{P}} = \underline{R}^{T} \underline{P} \qquad (rx1), \ \underline{R} = [\underline{\underline{U}}^{(1)}] \dots \ \underline{\underline{U}}^{(r)}] (nxr) \qquad (18)$$

Fox and Miura indicated this is equivalent to applying the Ritz-Galerkin principle in the subspace spanned by the set of vectors  $\underline{\mathbf{U}}^{(1)} \dots \underline{\mathbf{U}}^{(r)}$ . Although suggestions were provided, a systematic approach to calculating the basis vectors and deciding the value of r was not indicated. Noor and Lowder (8) suggested the use of the following as basis vectors,

$$\underline{\underline{U}}^{(1)} = \underline{\underline{U}}, \quad \underline{\underline{U}}^{(i+1)} = \frac{\partial \underline{\underline{U}}}{\partial \underline{d_i}} \qquad i=1,2,...D$$
 (19)

where D is the number of design parameters. Excellent results were obtained.

### Exact Methods

Successive Modification Methods (SMM)

This method, developed by Kavlie and Powell (7), incorporates the modification matrix,  $\Delta K$ , a single <u>non-null</u> column at a time. The basis of this approach is the Sherman-Morrison formula (see Householder (5)) for the inverse of a modified matrix, i.e.

$$(\underline{A} + \underline{X} \underline{Y}^{T})^{-1} = \underline{A}^{-1} - (\underline{A}^{-1} \underline{X})(\underline{Y}^{T} \underline{A}^{-1})/(1 + \underline{Y}^{T} \underline{A}^{-1} \underline{X})$$
(20)

where  $\underline{X}$  and  $\underline{Y}$  are column vectors representing the modification to  $\underline{A}$ .

The system's static equilibrium equations, before and after structural modifications are,

$$K U = P, (K + \Delta K) \bar{U} = P$$
 (nx1) (21)

Consider the special case when just column j of  $\Delta K$  is non-null. Designate this column as  $X = \Delta K$ . Then

$$\underline{\Delta K} = \underline{X} \ \underline{Y}^{T} = \underline{\Delta K} \ \underline{Y}^{T}, \quad \underline{Y}^{T} = [0 \dots 0 \ 1 \ 0 \dots 0]$$

$$\underline{\Delta K} = \underline{X} \ \underline{Y}^{T} = \underline{A} \ \underline{X} \ \underline{Y}^{T} = [0 \dots 0 \ 1 \ 0 \dots 0]$$
(22)

and equation (20) implies

$$(K + \Delta K)^{-1} = K^{-1} - K^{-1} \times Y^{T} K^{-1} / (1 + Y^{T} K^{-1} X)$$
 (23)

To apply equation (23) directly would explicitly require  $\underline{K}^{-1}$ . Alternatively, post multiplying this equation by  $\underline{P}$  yields

$$\bar{\mathbf{U}} = \mathbf{U} - \mathbf{W} \mathbf{Y}^{\mathbf{T}} \mathbf{U}/(1 + \mathbf{Y}^{\mathbf{T}} \mathbf{W}), \qquad \mathbf{K} \mathbf{W} = \mathbf{X}$$
 (24)

where  $\underline{W}$  is calculated by back substitution using the triangularized form of  $\underline{K}$  from the unmodified system's solution.

For the more general case place the n non-null columns of  $\Delta K$  in the columns of the nxn matrix X. The non-null column numbers of  $\Delta K$  are designated by the set of ascending integers,

$$j_1 \ j_2 \cdots \ j_{n_c} \tag{25}$$

With these definitions the ith row of the n xn, Boolean matrix  $\underline{Y}^T$  which expands  $\underline{X}$  into  $\Delta K$  is null except for the number one (1) in column  $j_i$ . The stiffness modification matrix is then given by,

$$\frac{\Delta K}{(n \times n)} = \frac{X}{(n \times n_c)} \frac{Y^T}{(n_c \times n)} = \frac{\tilde{\Delta K}}{X} \frac{Y^T}{X}$$
(26)

For example consider a 10 dof system with modifications to  $\underline{K}$  in columns 2, 5, 7 and 9, then

$$n_{c} = 4, j_{1} = 2, j_{2} = 5, j_{3} = 7, j_{4} = 9$$

$$\underline{\Delta K} = [\underline{0} \mid \underline{\Delta K}_{2} \mid \underline{0} \mid \underline{0} \mid \underline{\Delta K}_{5} \mid \underline{0} \mid \underline{\Delta K}_{7} \mid \underline{0} \mid \underline{\Delta K}_{9} \mid \underline{0}]$$

$$\underline{X} = [\underline{\Delta K}_{2} \mid \underline{\Delta K}_{5} \mid \underline{\Delta K}_{7} \mid \underline{\Delta K}_{9}] = [\underline{X}_{1} \mid \underline{X}_{2} \mid \underline{X}_{3} \mid \underline{X}_{4}] = \underline{\Delta K}$$

Then if more than one column of  $\underline{K}$  is modified, the <u>successive modification method</u> of Kavlie and Powell is employed. This assumes the following recursive form;

$$\underline{U}_{i} = \underline{U}_{i-1} - \underline{W}_{i}^{(i-1)} \underline{Y}_{i}^{T} \underline{U}_{i-1} / (1 + \underline{Y}_{i}^{T} \underline{W}_{i}^{(i-1)})$$
 (27)

$$\underline{\underline{W}}_{i}^{(k)} = \underline{\underline{K}}_{k}^{-1} \underline{\underline{X}}_{i} , \quad \underline{\underline{U}}_{o} = \underline{\underline{U}}$$
 (28)

where  $\underline{U}_{\cdot}$  is the system's displacement vector after the first i non-null columns of  $\underline{\Delta K}$  are considered. Note that  $\underline{W}_{\cdot}^{(K)}$  is the system's displacement vector at the stage when k non-null columns of  $\underline{\Delta K}$  are included, with  $\underline{X}_{\cdot}$  as the external load vector. These vectors are calculated by the formulas

$$\underline{W}_{i}^{(k)} = \underline{W}_{i}^{(k-1)} - \underline{W}_{k}^{(k-1)} \underline{Y}_{k}^{T} \underline{W}_{i}^{(k-1)} / (1 + \underline{Y}_{k}^{T} \underline{W}_{k}^{(k-1)})$$
(29)

$$\underline{K} \ \underline{W}_{i} = \underline{X}_{i} \qquad , \qquad i = 1, 2, \ldots n_{c}$$
 (30)

where  $\underline{W}_{1}^{(o)}$  is calculated by back substitution using the triangularized form of K from the unmodified system's solution. Equations (27) through ( $\overline{30}$ ) follow directly from the version of the Sherman-Morrison formula given by equation (24). A flow diagram for the 10 dof-n<sub>c</sub>=4 system is shown in Figure 1.

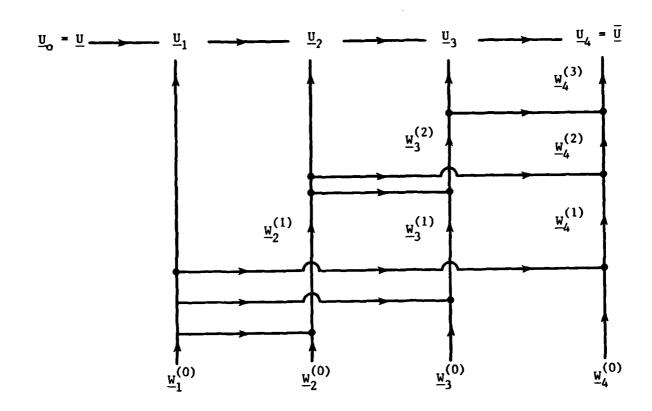


Figure 1. Flow diagram for static reanalysis (SMM), 10 dof -  $n_c$  = 4 system

Kavlie and Powell provide operation counts for this method, and emphasize that any reanalysis scheme should be carefully scrutinized as to when it is more efficient than complete reanalysis.

Direct Modification Method

This method was developed by Argyris, et. al.  $(\underline{6})$ . The derivation is based on the Woodbury formula (ref., 5),

$$(\underline{A} + \underline{E} \ \underline{C} \ \underline{F}^{T})^{-1} = \underline{A}^{-1} - \underline{A}^{-1} \ \underline{E} \ (\underline{C}^{-1} + \underline{F}^{T} \ \underline{A}^{-1} \ \underline{E})^{-1} \ \underline{F}^{T} \ \underline{A}^{-1}$$
 (31)

however a derivation from physical arguments is also included. For reanalysis of the static equilibrium equation substitute

$$\underline{A} = \underline{K}, \quad \underline{E} = \underline{b}^{T}, \quad \underline{F} = \underline{b}^{T}, \quad \underline{C} = \underline{\Delta K}$$
 (32)

$$\underline{\Delta K} = \underline{b}^{T} \underline{\tilde{\Delta K}} \underline{b} = \underline{E} \underline{C} \underline{F}^{T}$$
(33)

into equation (31) where  $\underline{b} = \underline{Y}^{T}$  (see equation (26)). This yields

$$(\underline{K} + \underline{\Delta K})^{-1} = (\underline{K} + \underline{b}^{T} \underline{\Delta K} \underline{b})^{-1} = \underline{K}^{-1} - \underline{K}^{-1} \underline{b}^{T} (\underline{\Delta K}^{-1} + \underline{b} \underline{K}^{-1} \underline{b}^{T})^{-1} \underline{b} \underline{K}^{-1}$$
(34)

Post multiply equation (34) by P and use equation (21). This gives

$$\underline{\Delta U} = \underline{\overline{U}} - \underline{U} = -(\underline{L}^{T})^{-1} \underline{Z} \underline{Q}^{-1} (\underline{Q}^{-1} + \underline{\Delta K})^{-1} \underline{\Delta K} \underline{b} \underline{U}$$
 (35)

where from the unmodified system solution the Cholesky decomposition of  $\underline{K}$  is,

$$K = L L^{T}$$
 (36)

and 
$$\underline{L} \underline{Z} = \underline{b}^{T}$$
,  $\underline{Q} = \underline{Z}^{T} \underline{Z}$  (37)

Computationally, the following procedure was suggested,

- (1) Compute  $r' = \Delta K$  b U, (2) Evaluate Q and Z,
- (3) Triangularize Q;  $Q = \underline{L}_Q \underline{L}_Q^T$  and evaluate  $Q^{-1}$ ,
- (4) Triangularize  $(Q^{-1} + \Delta K)$ ;  $(Q^{-1} + \Delta K) = L_s L_s^T$
- (5) Compute  $\underline{g}$  and  $\underline{\underline{r}}$ , from  $\underline{L}_{\underline{s}}$   $\underline{L}_{\underline{s}}^{\underline{T}}$   $\underline{g}$  =  $\underline{\underline{r}}$ , and  $\underline{L}_{\underline{O}}$   $\underline{L}_{\underline{O}}^{\underline{T}}$ ,  $\underline{\underline{r}}$ ,  $\underline{s}$ ,
- (6) Compute  $\underline{\mathbf{r}}$ ,  $= \underline{\mathbf{z}} \underline{\mathbf{r}}$ , finally
- (7) Solve  $L^{T}$  ( $\overline{U} U$ ) =  $-\tilde{r}$ ,...

Argyris emphasized that the preceeding algorithm is more efficient than complete reanalysis only when n is approximately less than 0.75B. Also mentioned is that Argyris's approach is generally more efficient than Kavlie and Powell's as expressed by equation (27)-(30).

## A New Method Pseudo Load Method

It is hypothesized here that the modifications actually behave as "closed-loop" deflection dependent loadings, forcing the unmodified system. Furthermore, suppose that for a localized modification, the added pseudo loading only is dependent on a small portion of the displacements. The modified system solution employs the available information from the solution of the unmodified system. This permits the modified system displacements to be expressed as functions of the displacements associated with only that portion of the system where modifications occur. Partitioning the modified system displacement vector and equating the modification related displacements yields a reduced set of equations. The following analytically summarizes the preceeding concepts.

Substitute equation (26) into equation (11)

$$\underline{K} \ \underline{\overline{U}} = \underline{P} - \underline{X} \ \underline{Y}^{T} \ \underline{\overline{U}}$$
 (nx1)

$$\underline{\overline{U}} = \underline{U} - \underline{K}^{-1} \underline{X} \underline{Y}^{\overline{T}} \underline{\overline{U}}$$
 (nx1)

Employ the previous notation of equation (24). Equation (39) is expressed as

$$\underline{\overline{U}} = \underline{U} - \underline{W} \underline{Y}^{T} \underline{\overline{U}}$$
 (nx1)

The Boolean matrix  $\underline{\underline{y}}^T$  condenses  $\underline{\bar{\underline{U}}}$  to the form,

$$\hat{\underline{\underline{U}}} = \underline{\underline{Y}}^{T} \underline{\underline{U}} \tag{41}$$

This condensed displacement vector contains only the coordinates which correspond to non-null columns of  $\Delta K$ , i.e.  $\bar{U}_j$ ,  $i=1,2,\ldots n_c$ . Equation (40) becomes,

$$\bar{\mathbf{U}} = \mathbf{U} - \mathbf{W} \hat{\bar{\mathbf{U}}} \tag{nx1}$$

By shifting rows  $(j_1, i=1,2,...,n_c)$  in  $\underline{\overline{U}}$ ,  $\underline{U}$  and  $\underline{W}$  to the top in equation (42), it is seen that

$$\begin{bmatrix} \hat{\underline{U}} \\ n_{c} \times 1 \\ \overline{\underline{U}}_{r} \\ (n-n_{c}) \\ \times 1 \end{bmatrix} = \begin{bmatrix} \hat{\underline{U}} \\ n_{c} \times 1 \\ \overline{\underline{U}}_{r} \\ (n-n_{c}) \\ \times 1 \end{bmatrix} - \begin{bmatrix} \hat{\underline{W}} \\ n_{c} \times n_{c} \\ \overline{\underline{W}}_{r} \\ (n-n_{c}) \\ \times n_{c} \end{bmatrix} \begin{bmatrix} \hat{\underline{U}} \\ n_{c} \times 1 \end{bmatrix}$$

$$(n \times 1)$$

$$(43)$$

The condensed displacement vector is then calculable from

$$(\underline{I}_{n_{c}} + \underline{\hat{w}}) \ \underline{\hat{y}} = \underline{\hat{y}}$$
 (44)

After which the remaining displacements are found using

$$\underline{\overline{U}}_{r} = \underline{U}_{r} - \underline{W}_{r} \hat{\underline{\overline{U}}}$$
 (n-n<sub>c</sub>)x1 (45)

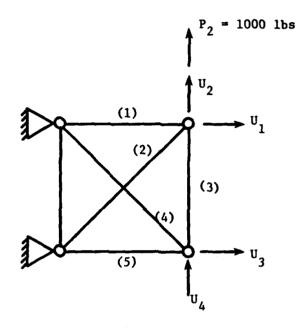
The computation of  $\underline{W}$  in equation (40) proceeds by solving the separate sets of equations  $\underline{K} \underline{W}_i = \underline{X}_i$ , i=1...n. These sets are solved by back substitution utilizing the triangularized form of  $\underline{K}$  from the unmodified system's solution.

The pseudo load method appears to resemble the direct modification method. Compared with the successive modification method, this method treats all modifications simultaneously. In addition, the derivation of the pseudo load method is simple and direct. The unmodified system's displacement vector and triangularized form of K may be obtained from any appropriate finite element code. A simple truss problem is included to illustrate the pseudo load method.

The geometric and material properties of the truss members are shown in Figure 2. Employing the direct stiffness method (ref. 11) with the appropriate boundary conditions provides the static equilibrium equation,

$$0.125 \times 10^{6} \begin{bmatrix} 5 & 1 & 0 & 0 \\ 1 & 5 & 0 & -4 \\ 0 & 0 & 5 & -1 \\ 0 & -4 & -1 & 5 \end{bmatrix} \begin{bmatrix} U_{1} \\ U_{2} \\ U_{3} \\ U_{4} \end{bmatrix} = \begin{bmatrix} 0 \\ 1000 \\ 0 \\ 0 \end{bmatrix}, \quad \underline{K} \ \underline{U} = \underline{P} \quad (46)$$

The Cholesky decomposition of  $\underline{K}$  is  $\underline{K} = \underline{L} \underline{L}^T$ ;



$$L_1 = L_3 = L_5 = 20 \text{ in.}$$
 $L_2 = L_4 = 20 \text{ $\sqrt{2}$ in.}$ 
 $A_1 = A_3 = A_5 = 1.0 \text{ in}^2$ 
 $A_2 = A_4 = \sqrt{2} / 2 \text{ in}^2$ 
 $E = 1.0 \times 10^7 \text{ lb/in}^2$ 

Figure 2. Truss used to illustrate the pseudo load method of static reanalysis

$$\underline{L} \ \underline{L}^{T} = 0.125 \times 10^{6} \begin{bmatrix} (5)^{\frac{1}{2}} & 0 & 0 & 0 \\ (5)^{-\frac{1}{2}} & (24/5)^{\frac{1}{2}} & 0 & 0 \\ 0 & 0 & (5)^{\frac{1}{2}} & 0 \\ 0 & -(10/3)^{\frac{1}{2}} & -(5)^{-\frac{1}{2}} & (22/15)^{\frac{1}{2}} \end{bmatrix} \\
\times \begin{bmatrix} (5)^{\frac{1}{2}} & (5)^{-\frac{1}{2}} & 0 & 0 \\ 0 & (24/5)^{\frac{1}{2}} & 0 & -(10/3)^{\frac{1}{2}} \\ 0 & 0 & (5)^{\frac{1}{2}} & -(5)^{-\frac{1}{2}} \\ 0 & 0 & 0 & (22/15)^{\frac{1}{2}} \end{bmatrix}$$

The solution for U proceeds in the standard fashion,

$$\underline{P} = \underline{L} \ \underline{L}^{T} \ \underline{U} \Longrightarrow \underline{P} = \underline{L} \ \underline{Y} \quad , \quad \underline{Y} = \underline{L}^{T} \ \underline{U}$$

$$\underline{U} = (2/11) \times 10^{-3} [-6 \quad 30 \quad 5 \quad 25]$$
(47)

Now suppose that the cross-sectional area of member 4 is changed so that  $A_4 \Longrightarrow A_4' = \sqrt{2}/2$  (1 +  $\alpha$ ). The responses of the modified system due to  $\underline{P}$ , as a function of  $\alpha$ , can be calculated by the pseudo load method. The required modification matrices and indices for the problem are,

$$\frac{\hat{\underline{v}}}{\underline{v}} = \begin{bmatrix} \bar{v}_3 \\ \bar{v}_4 \end{bmatrix}, \quad \underline{\underline{v}}_r = \begin{bmatrix} \bar{v}_1 \\ \bar{v}_2 \end{bmatrix}$$

$$e = 0.125 \times 10^6 \cdot \alpha, \quad n_c = 2, \quad j_1 = 3, \quad j_2 = 4$$

Using the decomposed form of  $\underline{K}$ , the matrix  $\underline{W}$  is,

$$|\underline{w}| = |\underline{w}_1| |\underline{w}_2| = \frac{\alpha}{11} \begin{vmatrix} 1 & -1 \\ -5 & 5 \\ 1 & -1 \\ -6 & 6 \end{vmatrix}$$

The modified system responses are from equations (44) and (45)

$$\bar{U}_3 = ((1 + 6\alpha/11)U_3 + \alpha U_4/11)/(1 + 7\alpha/11)$$
 (48)

$$\bar{U}_{4} = (6\alpha U_{3}/11 + (1 + \alpha/11)U_{4})/(1 + 7\alpha/11)$$
(49)

$$\bar{U}_1 = U_1 - \alpha (\bar{U}_3 - \bar{U}_4)/11$$
 (50)

$$\bar{U}_2 = U_2 - 5\alpha(-\bar{U}_3 + \bar{U}_4)/11$$
 (51)

Results like equations (48) and (49) are generally very useful since they provide (1) an efficient basis for performing a parametric study of the static response, and (2) means for prescribed static responses. The second use is illustrated by prescribing either  $\bar{U}_3$  or  $\bar{U}_4$  in equations (48) or (49) and then calculating the required factor  $\alpha$ , i.e. from equation (48),

$$\alpha = 11(\mathbf{U}_3 - \bar{\mathbf{U}}_3)/(7 \, \bar{\mathbf{U}}_3 - 6 \, \mathbf{U}_3 - \mathbf{U}_4) \tag{52}$$

For example if it is desired for  $\bar{U}_3=0.9~U_3$ , then from equation (52),  $\alpha=-0.234$ , and from equations (48)-(51),

$$\overline{U} = [-6.5 \ 32.5 \ 4.5 \ 28.0] \times (2/11) \times 10^{-3}$$

If from equation (52),  $\alpha$  < - 1.0, then the prescribed value of  $\bar{U}_3$  is not realizable by only modifying member 4.

# 4. CONCLUSIONS AND RECOMMENDATIONS

A new method for the nonlinear transient response of structures, in particular, submerged shell structures, is formulated. This is a modal superposition technique using residue flexibility in which the static response at each time is computed using an efficient static reanalysis technique.

This preliminary development of the method appears to indicate that this approach holds promise. Some numerical tests, which are not reported here, confirm this. The method seems to deserve further development.

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